

OUTLOOK

How calculus makes the modern world work

Review by Margaret Wertheim

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Margaret Wertheim is a science writer and author of books on the cultural history of physics. She is also an artist in the 2019 Venice Biennale, running May 11 to Nov. 24.

Toward the end of Steven Strogatz's marvelous new book on calculus, we learn how to calculate the speed of light using shredded cheese and a microwave oven. Take a plate and cover it lightly with a thin layer of cheese. Pop it into the oven after removing the machine's rotating base, so the cheese stands still, and give it a 30-second blast. The cheese will now manifest concentric rings of complete melting and little melting. Measure the distance between the rings. For a standard kitchen microwave, this should be around six centimeters. Double this to get the wavelength of the microwaves, about 12 centimeters, and multiply that by the frequency of the waves — usually found on the front of the device — about 2.45 billion cycles per second. Voilà, we get approximately 29.4 billion centimeters per second. And since all electromagnetic waves, including microwaves, radio waves and visible light, travel at the same velocity, we have deduced the speed of light, whose accepted value is 30 billion centimeters per second.

Strogatz, an acclaimed mathematician at Cornell who has also written a column for the New York Times and whose research papers on nonlinear dynamics are among the most-cited in academic science, tells this anecdote as part of a chapter describing the role of calculus in the development of technologies such as radar and CT scanning. In both, mathematical analysis of electromagnetic wave behavior was crucial.

This is one among a vast array of witty and astonishing stories Strogatz uses to illuminate how calculus has helped bring into being our contemporary world and so many of the instruments whose roles we now blithely assume.

Take the CT scanner. Who would have known that this lifesaving medical technique was funded indirectly by the Beatles? In the late 1960s a brilliant British engineer named Godfrey Hounsfield who had worked on radar and guided weaponry was given carte blanche by his employer to research whatever took his fancy. He developed a prototype of an X-ray-based device that he claimed could image soft tissues such as the brain and be used to see tumors, hemorrhages and blood clots.

At the time, all sensible scientists knew that X-rays could image only hard things, such as bone. His idea was dismissed as crackpot. Hounsfield tested his contraption on pigs' brains but couldn't get any medical people to take him seriously. "Finally," Strogatz writes, "one radiologist agreed to hear him out." At the end of the conversation the skeptical doctor handed Hounsfield a jar containing a human brain with a tumor and challenged him to image it. "Hounsfield soon brought back images of the brain that pinpointed not only the tumor but also areas of bleeding within it."

Radiologists were stunned, and others soon came on board, kicking off the age of computer-assisted tomography. Hounsfield's liberal employer was Electric and Musical Industries, or EMI, which had paid for this development with profits from a Liverpool band. In 1979, when Hounsfield was awarded the Nobel Prize for this work (along with South African physicist Allan Cormack), pop gold was transmuted into the gold of a Nobel medallion.

The invention of the CT scanner provided "another demonstration of the unreasonable effectiveness of mathematics," Strogatz writes, referencing a famous 1960 essay by physicist Eugene Wigner, which speculates on the mystery of why mathematics is found so widely in the material world.

Few branches of mathematics are so variously incarnated as calculus, which Strogatz deftly defines as a "sprawling collection of ideas and methods used to study anything — any pattern, any curve, any motion, any natural process, system or phenomena — that changes smoothly and continuously." That doesn't cover everything, but it encompasses a stupendous range of phenomena.

Calculus comes into play in general relativity, which describes the structure of space-time; it helps us analyze sound and music; and it's essential to the study of motion, for which Isaac Newton invented it. "Calculus enabled the creation of much of what made the global positioning system possible," Strogatz writes. It's used in identifying fingerprints and played a pivotal role in the development of the triple-drug therapy that turned HIV from a death sentence into a livable condition.

Richard Feynman, who applied tools of calculus in his theory of quantum electrodynamics, admiringly called it "the language God talks." Though perhaps a tad hyperbolic, Feynman's remark nonetheless provides a context for Strogatz's generous endeavor "to show calculus as a whole, to give a feeling for its beauty, unity, and grandeur."

This is a sweeping book that takes time and patience to read, but for anyone who's ever wanted to understand the essence of calculus and felt stymied by a hideous high school class, it is a richly rewarding experience. I would pair it with Jennifer Ouellette's delightful 2010 book, "The Calculus Diaries."

While the focus of Strogatz's book is on applied mathematics, in line with his position as a leading applied mathematician, among its finest qualities is its philosophical reflection on what calculus means. Here we come to the subject of the infinite, for at the heart of calculus is the idea of a minuscule infinity, a smallness so tiny it almost doesn't exist — what mathematicians refer to now as the infinitesimal. Like big-scale infinity, small-scale infinity has been one of the more controversial ideas in mathematics, and it has taken close to 2,000 years to rigorously define what the concept refers to.

In “Infinite Powers,” Strogatz articulates a credo of calculus: “to solve a hard problem about anything continuous,” slice it into infinitely many tiny parts, solve the tiny parts and then put them all together to solve the larger problem. He calls this the “Infinity Principle” and explains it in depth in several insightful chapters. What is perhaps most astonishing here is that the idea dates back to Archimedes, predating by more than 1,000 years Newton and Gottfried Wilhelm Leibniz’s formal invention of integral calculus in the 17th century.

Slicing and dicing has proved to be among the most powerful techniques in mathematicians’ tool box. As Strogatz notes: “By wielding infinity in just the right way, calculus can unlock the secrets of the universe.”

Infinite Powers

By Steven Strogatz

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